Mechanisms & Structures

– STRESS, STRAIN & FORCES

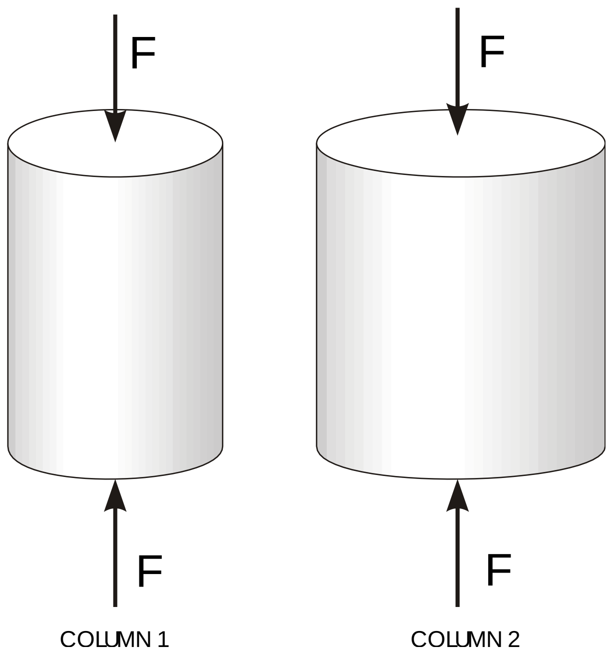
When mechanical or structural engineers are working on a project they need to select the right materials (wood, steel, iron, plastic etc). Two things they need to know are whether the materials they choose will be strong enough and whether they will stretch or compress too much when forces are applied.

When a force is applied to a part of a structure it will experience stress. A high level of stress will result in the part failing (breaking). Stress is calculated with the following formula –

$$Stress = \frac{Force}{Area}$$

$$\sigma = \frac{F}{A}$$

Look at the two parts below. The one on the left will be more likely to fail when the force is applied. This is because it has a smaller cross-section area so will experience more stress.



SM.H.O3.fig7

where Force is measured in Newtons (N) and Area is the cross-sectional area measured in mm^2. Stress therefore is measured in Nmm^{-2} and is denoted by the Greek letter sigma (σ).

Worked examples: Stress

A square bar of 20 mm x 20 mm cross-section is subjected to a tensile load of 500 N. Calculate the stress in the bar.

$$Stress = \frac{Force}{Area}$$

$$\sigma = \frac{F}{A}$$

$$\sigma = \frac{500}{400}$$

$$\sigma = 1.25\mathrm{Nmm}^{-2}$$

Stress in the bar = 1.25 Nmm^{-2}

A column of section 0.25 m^2 is required to act as a roof support. The maximum allowable working stress in the column is 50 Nmm^{-2}. Calculate the maximum compressive load acting on the column.

$$Stress = \frac{Force}{Area}$$

$$Force = Stress \times Area$$

$$Force = 50 \times 0.25 \times 10^{6}$$

$$Force = 12.5MN$$

Maximum compressive load acting on the column = 12.5 MN

The stress in a steel wire supporting a load of 8 kN should not exceed 200 N/mm^2. Calculate the minimum diameter of wire required to support the load.

$$Stress = \frac{Force}{Area}$$

$$Area = \frac{Force}{Stress}$$

$$Area = \frac{8000}{200}$$

$$Area = 40mm^2$$

$$Area = \frac{\pi d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}}$$

$$d = \sqrt{\frac{4 \times 40}{\pi}}$$

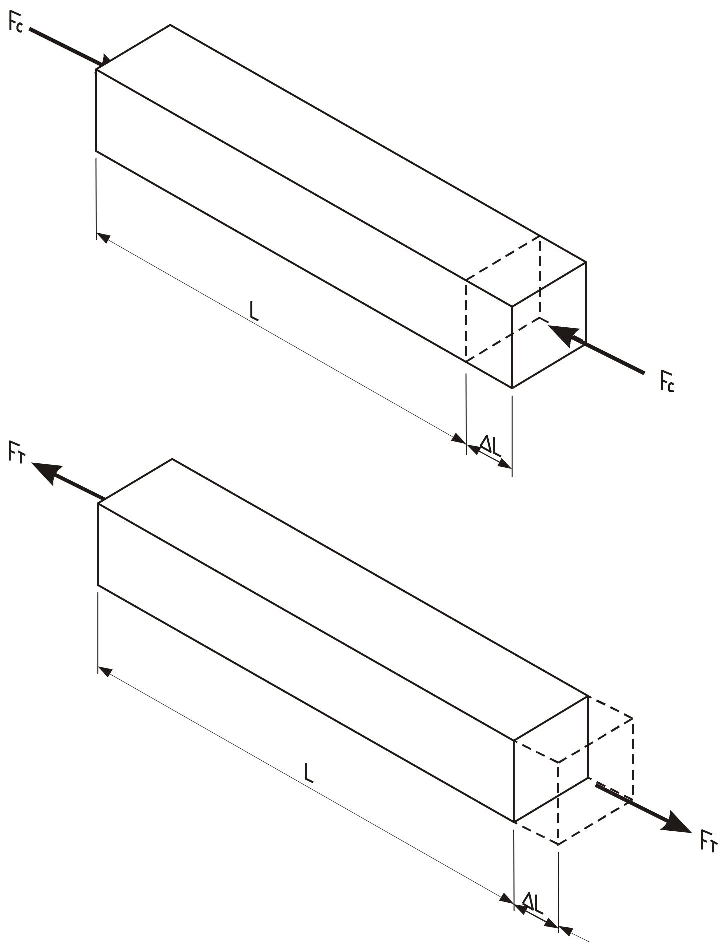
$$d = 7.14mm$$

Minimum diameter of wire required to support load = 7.14 mm

Strain

The result of applying a load or force to a structural member is a change in length. Every material changes shape to some extent when a force is applied to it. This is sometimes difficult to see in materials such concrete and we need special equipment to detect these changes.

If a compressive load is applied to a structural member, then the length will reduce. If a tensile load is applied, then the length will increase. This is shown in the diagrams below.



SM.H.O3.fig8

The result of applying a load to a structural member is called STRAIN. This is calculated using the formula:

$$Strain = \frac{Change\ in\ Length}{Original\ Length}$$

$$\varepsilon = \frac{\Delta L}{L}$$

where length in both cases is measured in the same units (m or mm). As the units cancel each other out, strain is dimensionless. This means that there are no units of strain. Put simply, strain is a ratio that describes the proportional change in length in the structural member when a direct load is applied. Strain is denoted by the Greek letter epsilon (ε).

Worked examples: Strain

1. A steel wire of length 5 m is used to support a tensile load. When the load is applied, the wire is found to have stretched by 2.5 mm. Calculate the strain for the wire.

$$\varepsilon = \frac{\Delta L}{L}$$

$$\varepsilon = \frac{2.5}{5000}$$

$$\varepsilon = 0.0005$$

Strain in the wire = 0.0005

2. The strain in a concrete column must not exceed 5 x 10-4. If the column is 3 m high, find the maximum reduction in length produced when the column is loaded.

$$\varepsilon = \frac{\Delta L}{L}$$

$$\Delta L = \varepsilon \times L$$

$$\Delta L = \left(5 \times 10^{-4}\right) \times 3000$$

$$\Delta L = 1.5mm$$

Reduction in length of column = 1.5 mm

Assignments: Stress and Strain

1. A bar of steel 500 mm long has a cross-sectional area of 250 mm2 and is subjected to a force of 50 kN. Determine the stress in the bar.

2. A wire 4 mm in diameter is subjected to a force of 300 N. Find the stress in the wire.

3. What diameter of round steel bar is required to carry a tensile force of 10 kN if the stress is not to exceed 14.14 N/mm2.

4. A wire 10 m long stretches 5 mm when a force is applied at one end. Find the strain produced.

5. A tow bar, 1.5 m long, is compressed by 4.5 mm during braking. Find the strain.

6. The allowable strain on a bar is 0.0075 and its length is 2.5 m. Find the change in length.

7. During testing, a shaft was compressed by 0.06 mm. If the resulting strain was 0.00012, what was the original length of the shaft?

8. A piece of wire 6 m long and diameter of 0.75 mm stretched 24 mm under a force of 120 N. Calculate stress and strain.

9. A mild steel tie-bar, of circular cross-section, lengthens 1.5 mm under a steady pull of 75 kN. The original dimensions of the bar were 5 m long and 30 mm in diameter. Find the intensity of tensile stress in the bar and determine the strain.

10. A mass of 2500 kg is hung at the end of a vertical bar, the cross-section of which is 75 mm x 50 mm. A change in length in the bar is detected and found to be 2.5 mm. If the original length of the bar is 0.5 m, calculate the stress and strain in the bar.

Vectors

Force is a vector quantity and has both magnitude and direction. This means it is often convenient to represent a force by a line, that is, a vector quantity, which is sometimes easier to understand visually. The direction of the force may be indicated by an arrow-headed line, with the length of the line drawn to scale to represent the size of the force. This line is called a vector.

Example

The cyclist pedalling with a force of 800 N is being assisted by a tail-wind of 400 N, but the friction from the road surface measures 200 N.

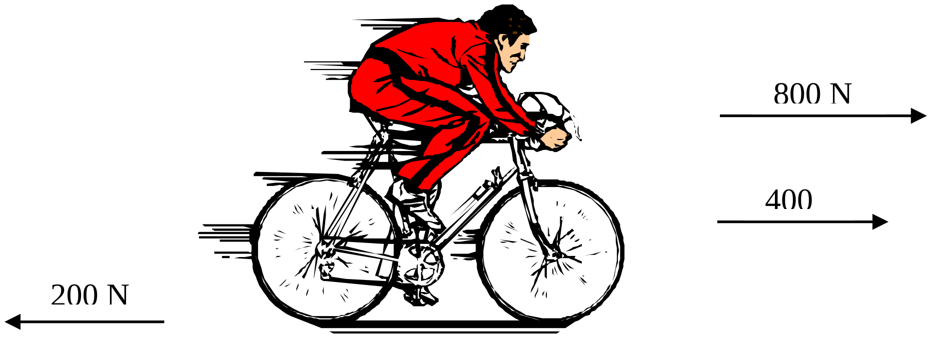


Figure 1

The overall effect will be 800 N + 400 N – 200 N = 1000 N (or 1 kN).

A suitable scale would be selected – possibly 10 mm to represent 20 N – and using this scale each force is drawn in turn, one following on from the other.

800 N + 400 N – 200 N = 1000 N or 1 kN

Figure 2: a vector diagram

When the three forces are added together, they can be replaced by a single force that has the same effect, called the ‘resultant’.

RESULTANT = 1 kN

Figure 3

Vectors are also used to find the resultant of two forces that are inclined at an angle to each other.

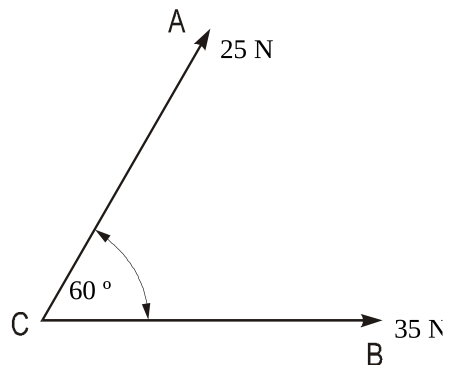


FIGURE 4

In the example above the resultant of the two forces can be found by drawing two vectors. First choose a suitable scale and draw the two vectors CA and CB.

SCALE: 10 MM = 10 N

25 mm = 25 N = CA
35 mm = 35 N = CB

The bigger the scale the more accurate the vectors.

From A draw a line parallel to CB, and from B draw a line parallel to CA. Call the point where the two lines intersect point D. Now draw a line from C to D. A line drawn from C to D is the resultant of the two forces CA and CB.

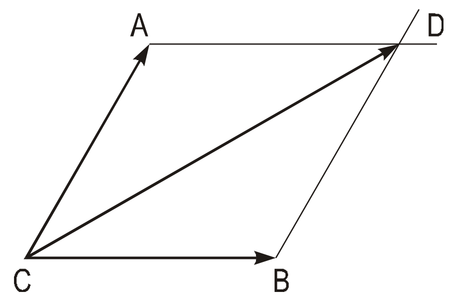


Figure 5

The resultant has a magnitude of 46 N by measurement.

Equilibrium

Certain conditions must apply within structures in order to create stability. The resultant is made up of the combined forces that are trying to move an object or structure in a set direction. If such a force were applied without an opposing force then major problems could occur. Structures have to remain in a stable or balanced state called 'equilibrium', which simply means 'balanced'. There are three types of balancing that must exist if structures, bodies, objects, etc. are to remain in equilibrium: horizontal, vertical and rotational forces must all balance.

The general conditions of equilibrium are as follows.

- upward forces = downward forces
- leftward forces = rightward forces
- clockwise moments = anticlockwise moments

Example 1

Consider again the same two forces in figure 5. Are they in equilibrium? It is easily seen that a force must be added acting downwards to the left, but we cannot tell from this exactly how large this force must be or its exact direction (figure 6).

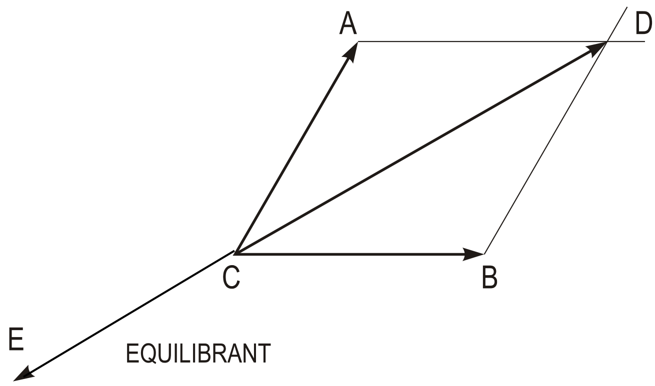


Figure 6

The resultant has been drawn and it can be seen that to balance it, the equilibrant CE is required. If the forces F_1 and F_2 are drawn as in figure 7 then it is much easier to obtain the equilibrant by completing the triangle, as shown in figure 7.

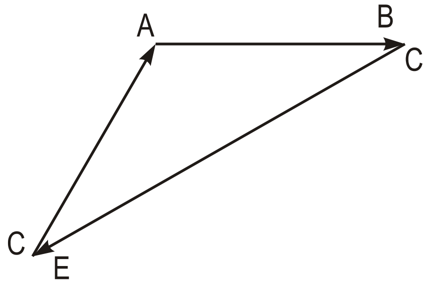


FIGURE 7

Example 2
A crane is fixed against a wall, as shown in figure 8. Find the forces in the compression and tension members.

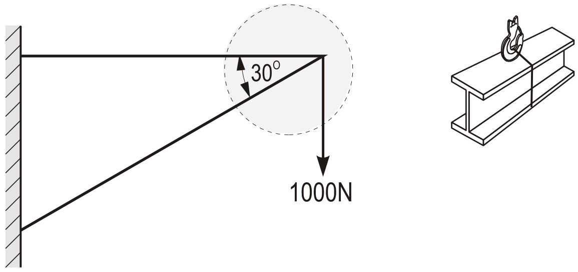


Figure 8

To find the forces created in the tension and compression members by the 1000 N load, draw the triangle from the area circled. Select a suitable scale and then draw the known force first, the 1000 N load (figure 8). A line is drawn through one end of the load line parallel to one of the unknown forces. Another line is drawn through the other end, parallel to the second unknown force. By measuring each line, the size of each force can be found. (Note: the arrowheads must follow round the triangle.)

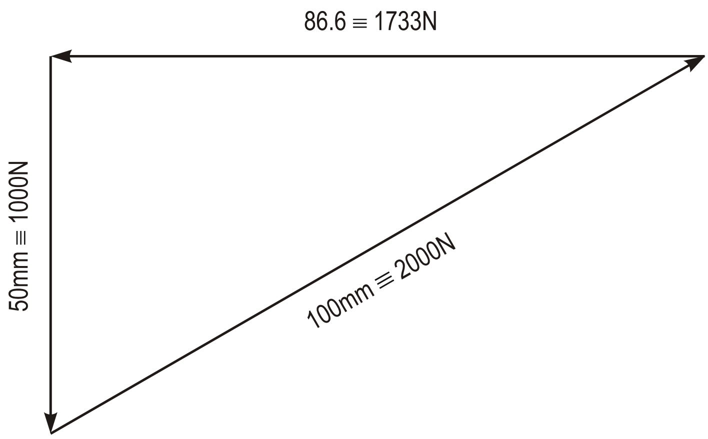


Figure 9

Scale: 10 mm = 200 N

THE COMPRESSION MEMBER = 2000 N

The tension member = 1733 N

Equilibrium: task 1

1. Study the following statements and cross out the incorrect answer.

   - A body that is accelerating is in a state of equilibrium.

     TRUE/FALSE

   - For a body to be in a state of equilibrium it is necessary only for the vector sum of the forces acting on it to be zero.

     TRUE/FALSE

   - A resultant force is a single force that can replace two or more forces.

     TRUE/FALSE

   - If two or more forces are replaced by a resultant force, the effect on the body is changed.

     TRUE/FALSE

   - An equilibrant force is the force that, if applied to a body, will cause the body to be in a state of equilibrium.

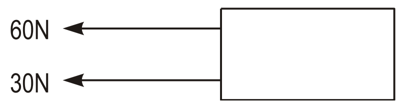
     TRUE/FALSE

   - The equilibrant force is identical to the resultant force.

     TRUE/FALSE

2. Try to explain two conditions necessary for a structure or body to be in equilibrium.

3. Two forces are acting on a body as shown.



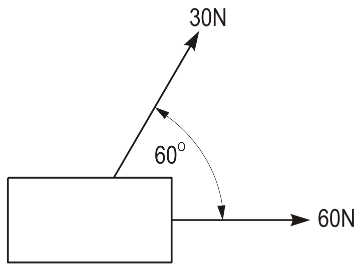
(a) Graphically indicate their size and direction.
(b) Graphically indicate the resultant of the two forces.

4. Two forces are acting on a body as shown.

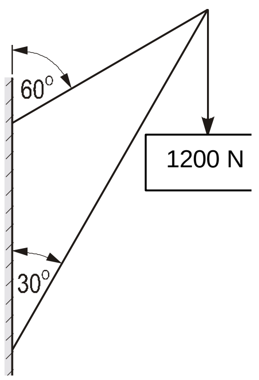
(a) Graphically indicate their size and direction.

(b) Calculate the resultant and direction of the two forces.

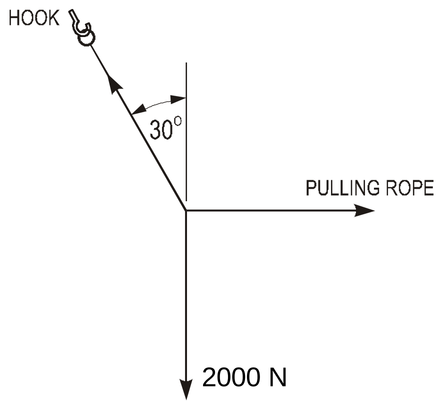
5. What are the resultant and equilibrant of the two forces affecting the system below?



6. A small crane is used on a fishing trawler to lift cases of fish to the dock. The weight of the lift is 1200 N. Determine the size and direction of the forces in each of the crane members. (Use a scale where 10 mm represents 200 N.)



7. A weight of 2000 N is suspended by a rope attached to a hook firmly fixed to a roof joist. A second rope is attached to the vertical rope and pulled horizontally until the rope makes an angle of 30° to the vertical as shown. Determine the horizontal pull on the rope and the force on the hook.



8. The figure below shows a cranked lever that is part of a gear-change mechanism. Find the resultant force F_R acting on the hinge pivot and the angle θ.